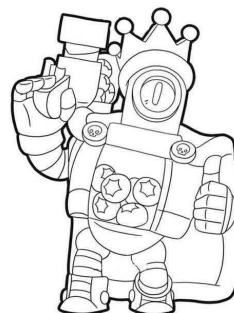# Theme Round

LMT Fall 2024
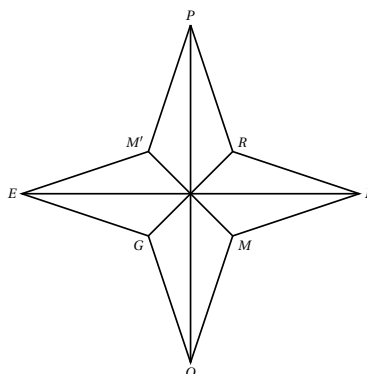
December 14, 2024

Gaming

At math team, the main activity is math but a concerning close second is gaming. Here are some of our favorite games.

1. **[6]** In Genshin Impact, $PRIMOGEM'$ is the octagon in the diagram below. Let A be the intersection of PO and IE. Suppose $PR = RI = IM = MO = OG = GE = EM' = M'P$, $AP = AI = AO = AE = 4$, and $AR = AM = AG = AM' = \sqrt{2}$. Find the area of $PRIMOGEM'$.

2. **[8]** In Pokemon, there are 10 indistinguishable Poke Beans in a pile. Pikachu eats a prime number of Poke Beans. Charmander eats an even number of Poke Beans. Snorlax eats an odd number of Poke Beans. Find the number of ways for the three Pokemon to eat all 10 Poke Beans.

3. **[10]** In Survev.io, Calvin observes that he has exactly twice as much blue ammo as red ammo. After firing one blue bullet and 9 red bullets, he remarks that the amount of blue ammo he has is divisible by 5 and the amount of red ammo he has is divisible by 7. Find the least amount of red ammo he could have started with.

4. **[12]** In Brawl Stars, Rico can shoot his opponent directly or make his bullet bounce off the wall at the same angle. His opponent is 15 feet in front of him and there are infinitely long walls 1 feet to the left and right of Rico. If Rico's bullet travels d feet before hitting the opponent, find the sum of all possible integer values of d.

5. **[14]** In Ace Attorney, Phoenix Wright is rolling a standard fair 20-sided die. He can roll this die up to three times. After each roll, Phoenix can yell "Objection!" to roll again, or "Hold It!" to stop and keep his current number.

   If Phoenix plays optimally to maximize his final number, find the expected value of this number.

Brainrot

What is up sigmas!? If your attention span has already elapsed feel free to scroll and start the problems. At this point, brainrot is everywhere: it is the Oxford Dictionary's 2024 word of the year, and here at LMT, certain members are known to say things like "sigma sigma on the wall" quite frequently.

1. **[6]** Suppose h, i, o are real numbers that satisfy the products $hi = 12$, $ooh = 18$, and $hohoho = 27$. Find the value of the product $ohio$.

2. **[8]** A positive n is called *sigma rizz* if the sum of its digits is equal to two times the number of digits it has. Find the number of sigma rizz numbers less than 1000.

3. **[10]** Let MEW and MOG be isosceles right triangles such that E, M, O are collinear in that order and G, M, W are collinear in that order. Suppose $ME = MW = \sqrt{6 - 4\sqrt{2}}$ and $MO = MG = \sqrt{6 + 2\sqrt{2}}$. Find the least possible area of a circle which contains both triangles MOG and MEW.

4. **[12]** Let S, K, I, B, D, Y be distinct integers from 0 to 9, inclusive. Given that they follow this equation:

$$
\begin{array}{ccccc}
 & S & K & I & B \\
- & I & D & I & D \\
\hline
 & & & D & Y \\
\end{array}
$$

find the maximum value of $\overline{SKIBIDI}$.

5. **[14]** Tnag is repeating the phrase "sigma sigma on the wall" an infinite number times. Between each word, there is exactly one second of pause. Adam has heard the phrase so many times that he has come up with a game using two numbers x and y: Start with a score of 0.

   - At a random time, Adam will hear the word a (each of the 5 words are equally likely to be heard).
   - Then
     - if a is "sigma", Adam will multiply his score by x, and
     - if a is any of the other words, Adam will add y to his score.

   Let $f(x, y)$ be Adam's expected score after infinitely many steps. Find

   $$\sum_{n=2}^{\infty} f\left(\frac{1}{n}, \frac{1}{n^2}\right).$$

Rappers

This year was a very eventful one for rap. Let's celebrate by going through what these rappers did in their spare time.

1. **[6]** Travis Scott says "FEIN" every 0.8 seconds. Find the tens digit of the number of times he says "FEIN" in 1 minute.

2. **[8]** Eminem is trying to find the real Slim Shady in a row of 2025 indistinguishable Slim Shady clones, one of which is the real Slim Shady. Eminem randomly guesses, and if he guesses wrong, a new clone joins the row and all the clones randomly rearrange themselves. He keeps guessing as more identical clones are added, trying to find the real Slim Shady. Find the probability that he will eventually find him within 15 guesses.

3. **[10]** Kendrick Lamar and Drake are cutting their circular beef to share with their fans. The cuts must pass all the way from one side of the beef to the other, and no other modifications may be performed on the beef (e.g. folding, eating, stacking, etc.). Find the minimum number of cuts they will need to split their beef into 2024 pieces.

4. **[12]** Let NAS be a triangle such that $NA = NS = 5$ and $AS = 6$. Let D be the foot of the altitude from N to AS and E the foot of the altitude from A to NS. Point X lies on line DE outside the triangle such that $XA = \frac{18}{5}$. Find XS.

5. **[14]** Kanye West's favorite positive integer this year is c, and last year it was $c - t = 20011$ (a prime), for some positive integer t relatively prime to c. His two most streamed albums got a and b streams this year and $a - t$ and $b - t$ streams last year with $a > b > c$. Suppose $a \leq 1.6 \times 10^9$ and his favorite integer in each year divides the number of streams for both albums in the corresponding year. Find the largest possible value of c.

Tiebreaker Estimation

This problem will only be used to break ties for individual aggregate awards. If two competitors are tied the one closest to the answer will win.

1. **[TIEBREAKER]** From the screen, to the ring, to the pen, to the king, KSI has played games, been in brainrot memes, and rapped. To honor him, estimate the number of times the letters K, S, I appear on this round (excluding answer sheet) in that order, potentially with letters in between them.